Uncertainty, probability and bad luck

Petter Abrahamsen

The Gathering, Sassenheim-Leiden

23.10.2023

Uncertainty is lack of knowledge

- Incomplete information
- Incomplete understanding

- Car or train?
  How is traffic today?

Uncertainty is the normal

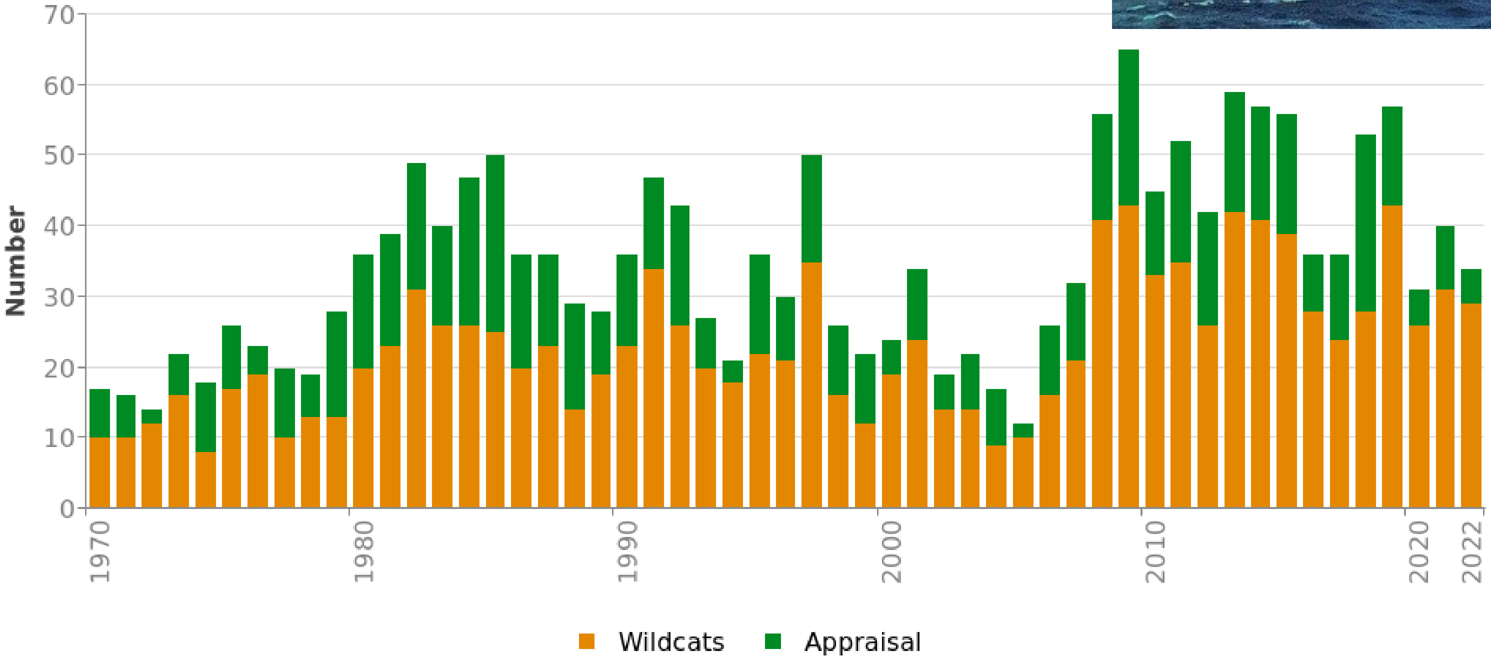
- Humans accept uncertainty
- We love opportunity
- We hate risk

Uncertainty is the norm

- Humans accept uncertainty
- We love opportunity
- We hate risk - not

Exploration

1230 wildcat wells on the NCS
122 fields have been in production



Exploration

Ocean Viking discovered Ekofisk in December 1969 after 37 dry wells

1230 wildcat wells on the NCS
122 fields have been in production

Probability for discovery: 122/1230 = 9,9 % (2022)
1/38 = 2,6 % (1969)

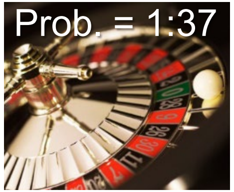
Probability is the mathematical formalism used for quantifying uncertainty

- Started with gambling
- Probability theory gives logically correct statements, given assumptions

Probability can be

- Empirical: Discovery rate = 2,6 % to 9.9 %

- From physics: 37 slots:



The time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

- Subjective: Chance of finding a runaway cat alive

Lets do some math (probability theory)

P(success) = 0.05 (5 %)
P(failure) = 1 – P(success) = 0.95

Probabilities can get really messy

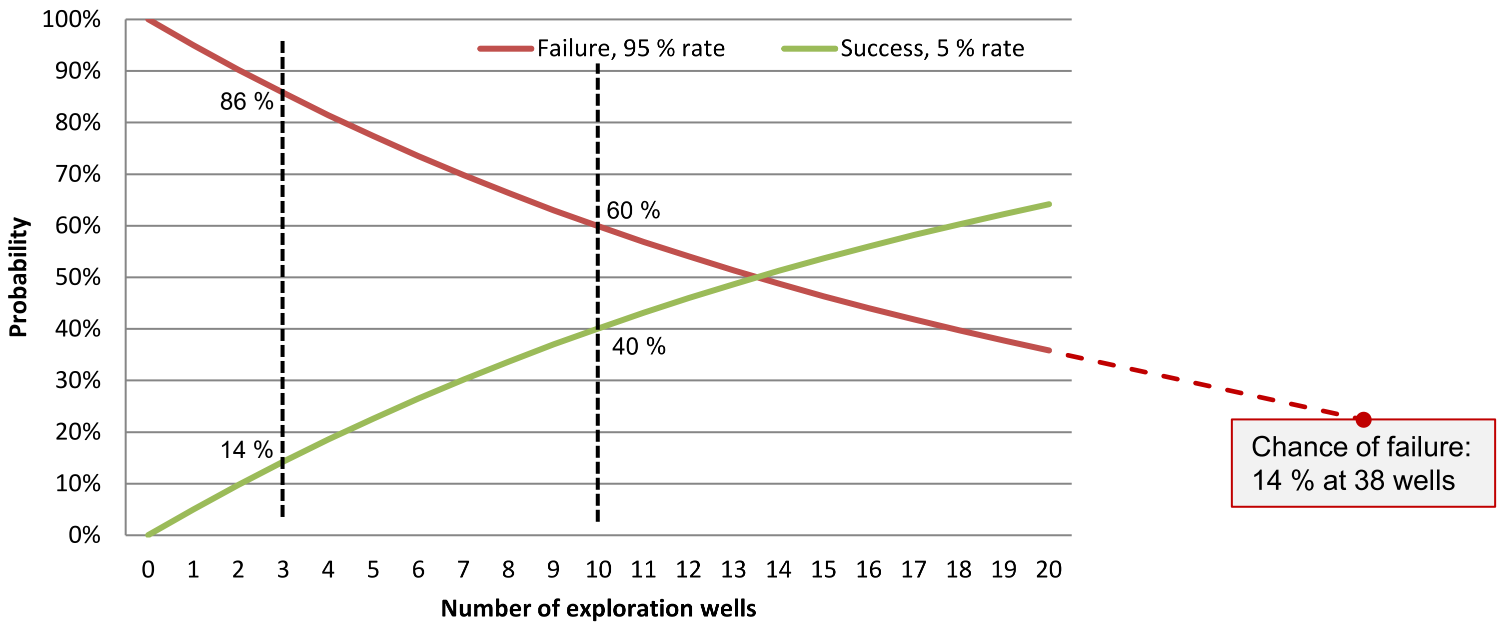
Probability for success 3 times in a row:
P(success)·P(success)·P(success) = 0.05^3 = 0.000125

Probability for failure 3 times in a row:
P(failure)·P(failure)·P(failure) = 0.95^3 = 0.857

What is this probability?: 1 – 0.857 = 0.143

The probability for 1 success (three possibilities)
+ the probability for 2 successes (three possibilities)
+ the probability for 3 successes (one possibility)

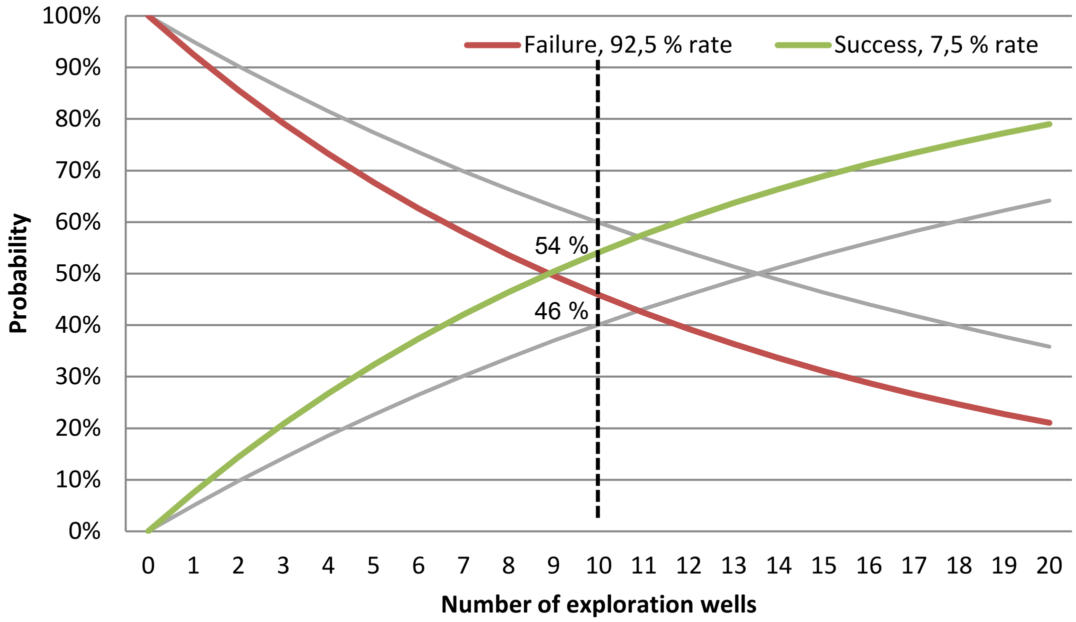
If there is a 5 % chance of finding an economical field by drilling a single well



After 10 wells: 40 % chance of success

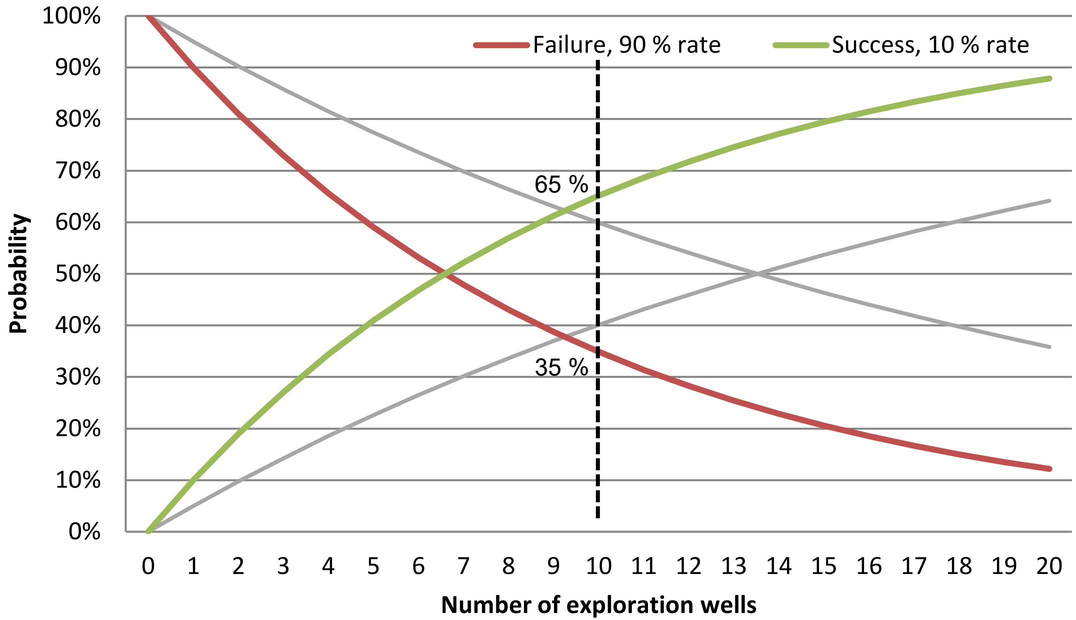
60 % chance of total failure

If there is a 7,5 % chance of finding an economical field by drilling a single well



After 10 wells: 54 % chance of success
46 % chance of failure

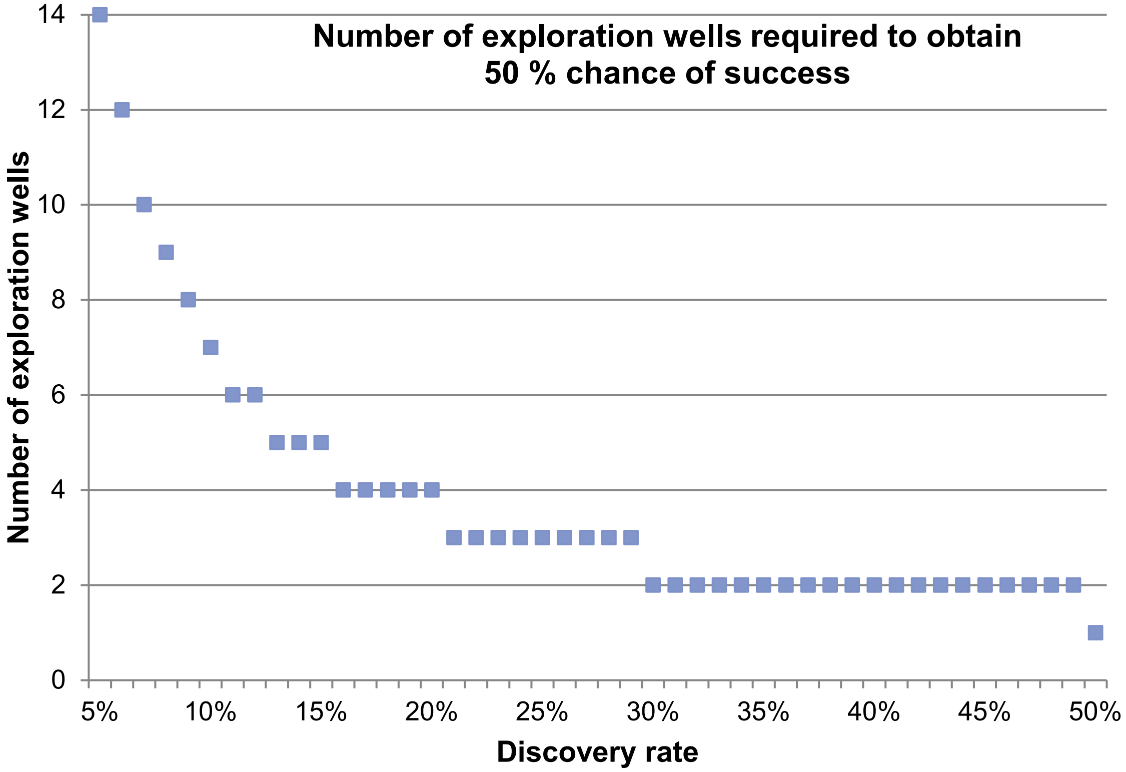
If there is a 10 % chance of finding an economical field by drilling a single well



After 10 wells: 65 % chance of success

35 % chance of failure

Improving the odds helps
(but it doesn't guarantee success

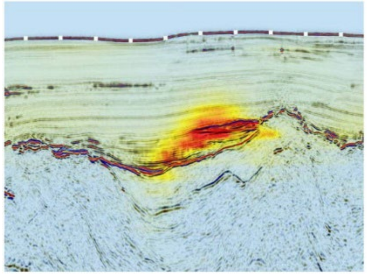


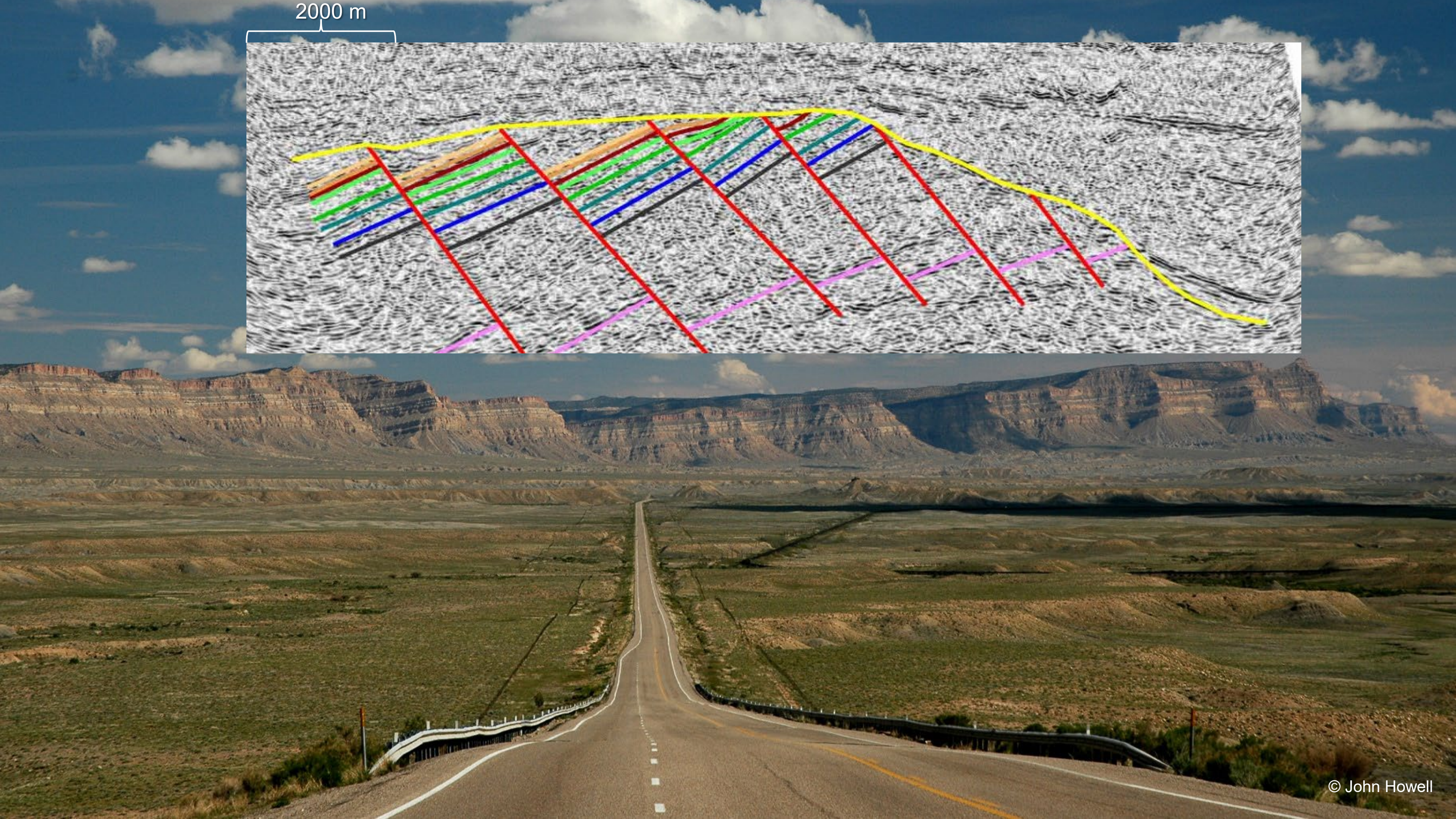
So what can we do to increase the odds?

1. Collect (more) information

2. Use available information better
   a) Processing
   b) Interpretation
   c) Integration

3. Make correct calculations ☺




2000 m

Seismic inversion

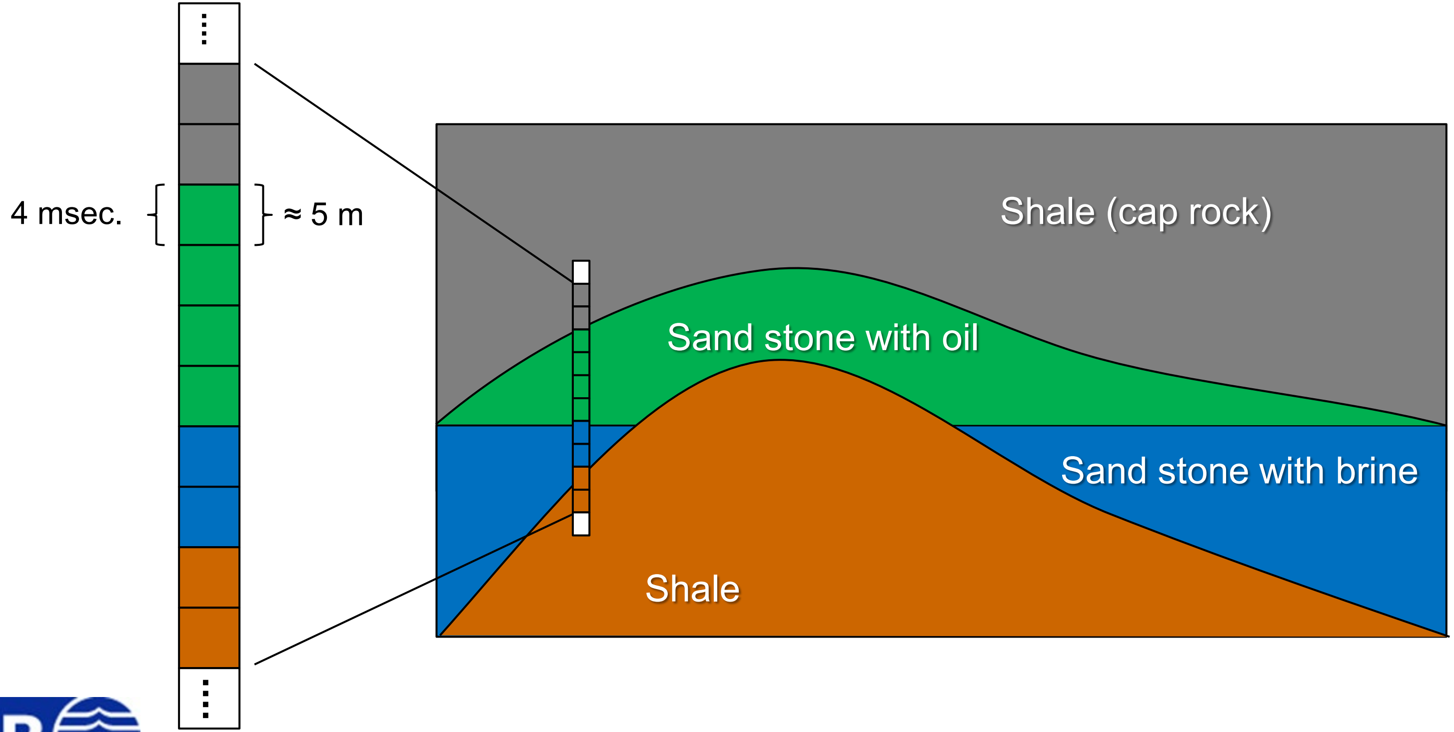
Straightforward inversion will not work!

This is why all inversion methods
include some form of ***regularization***

Why probabilistic inversion

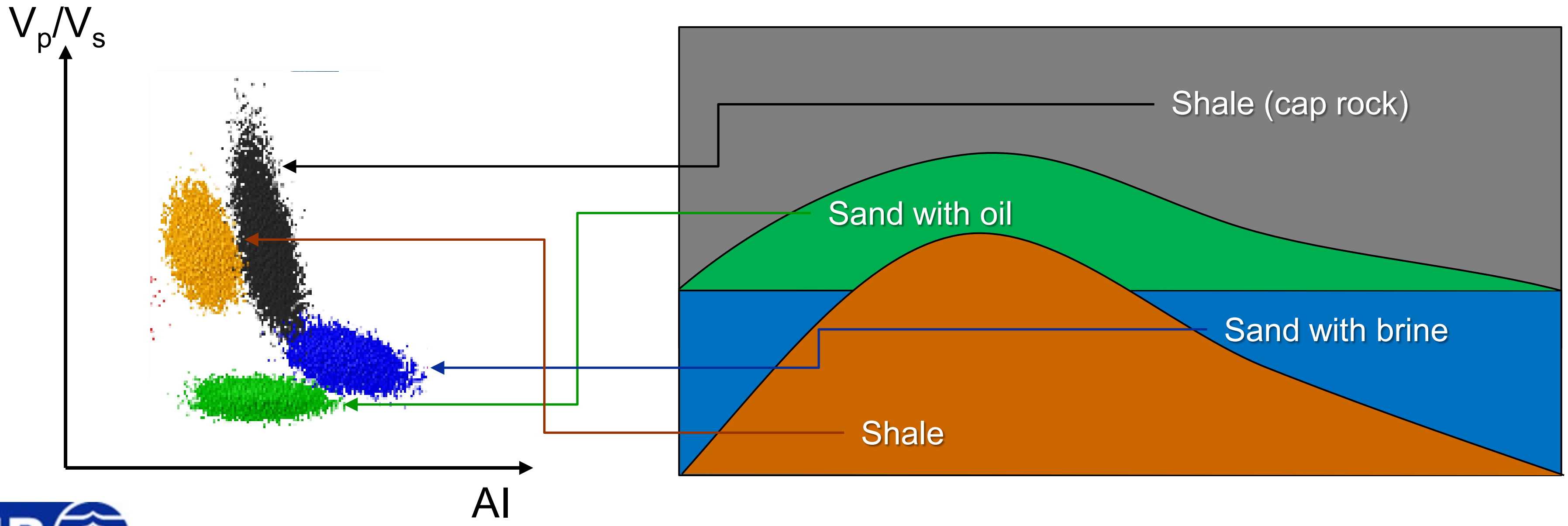
- Bayesian statistics combines **prior information** and **data** (angle stacks)
  - Constrain the solution space (regularization)
- Uncertainty is included in the model
  - Noise in seismic data (and model approximations)
  - Geologic variability
- Main output
  - Updated **probabilities** for lithology and fluid classes (LFCs) in the inversion cube

Categorization of the subsurface

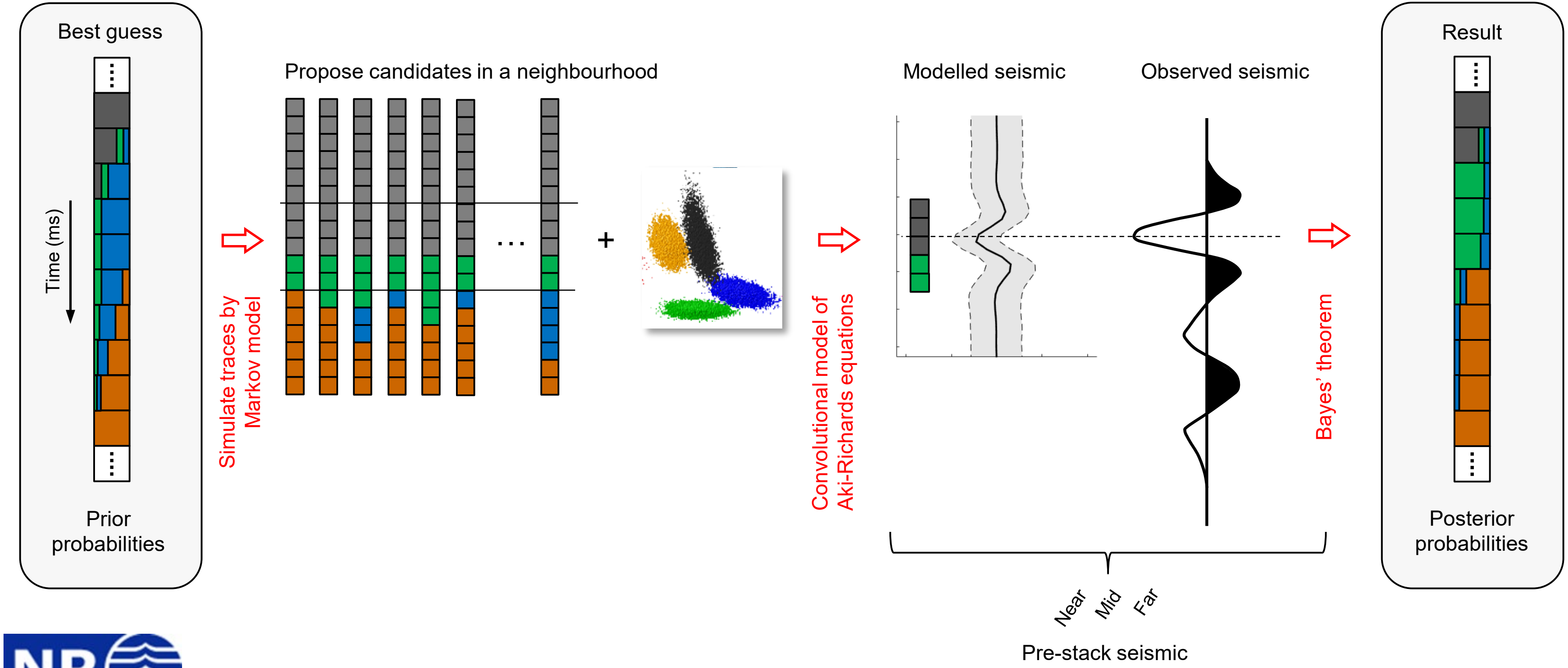


Elastic properties of the categories

- Separate regions in V_p, V_s and ρ space define different lithology and fluid classes (LFCs)



Estimate the probability



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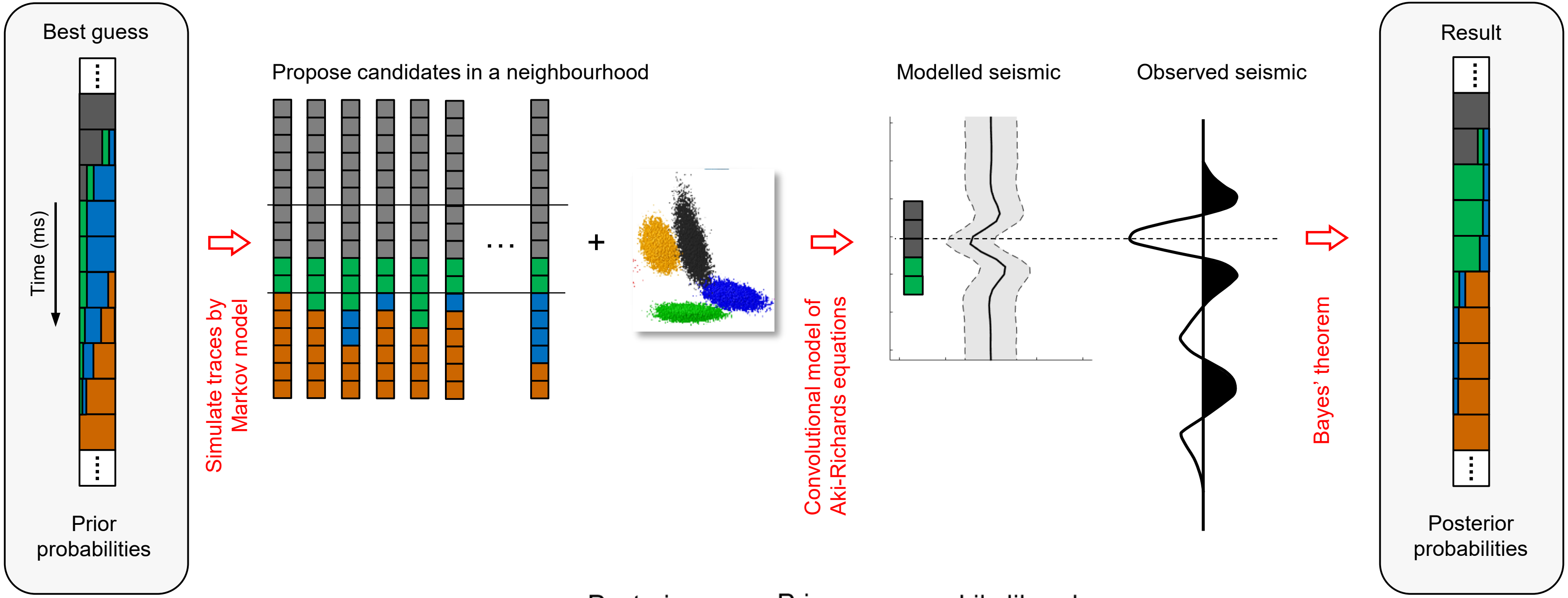
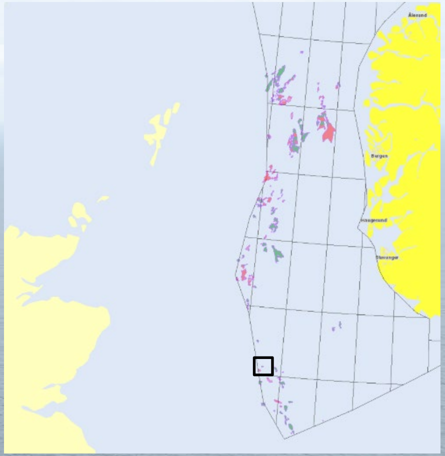


	Posterior		Prior		Likelihood
Bayes rule	$p(f\|d)$	$\propto$	$p(f)$	$*$	$p(d\|f)$

The Volund field

Tertiary deep-marine reservoir in the North Sea

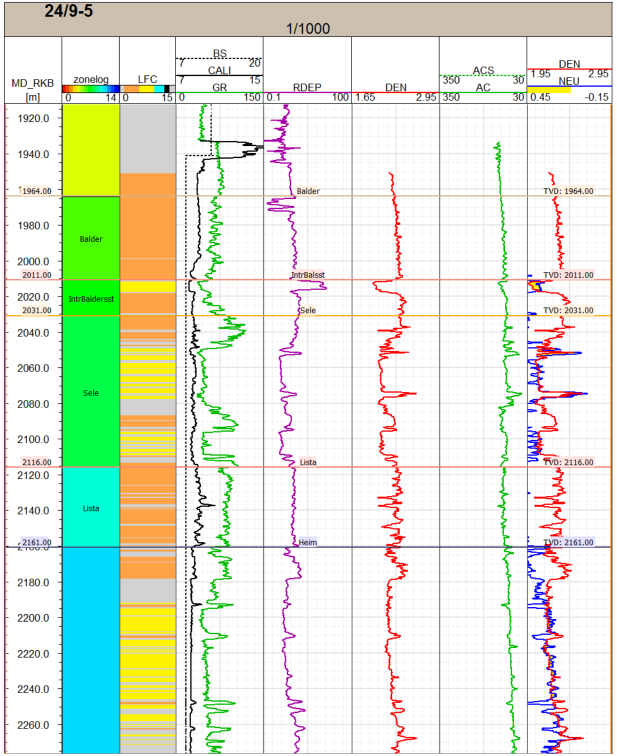


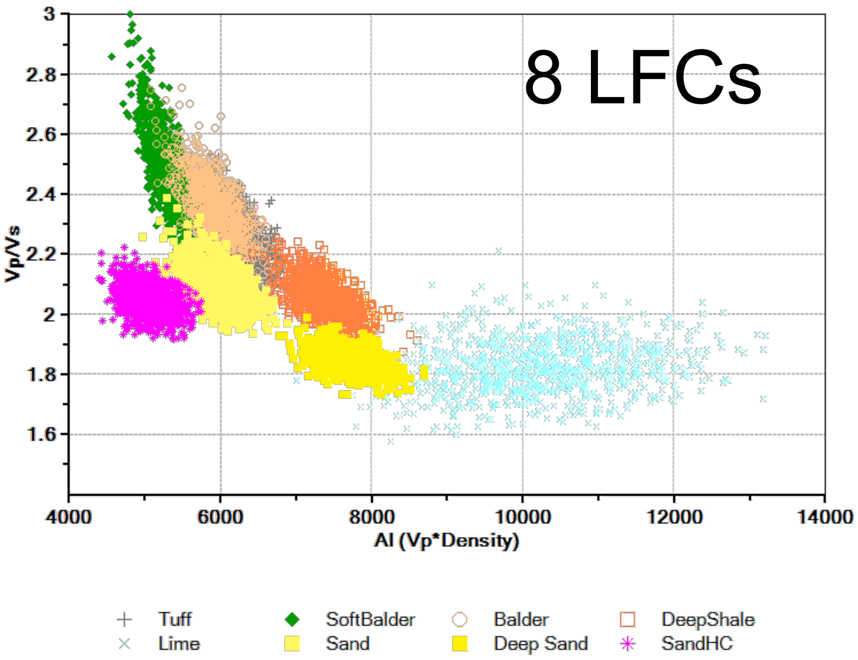
(maps from npd.no)

Injected sand dykes and sills

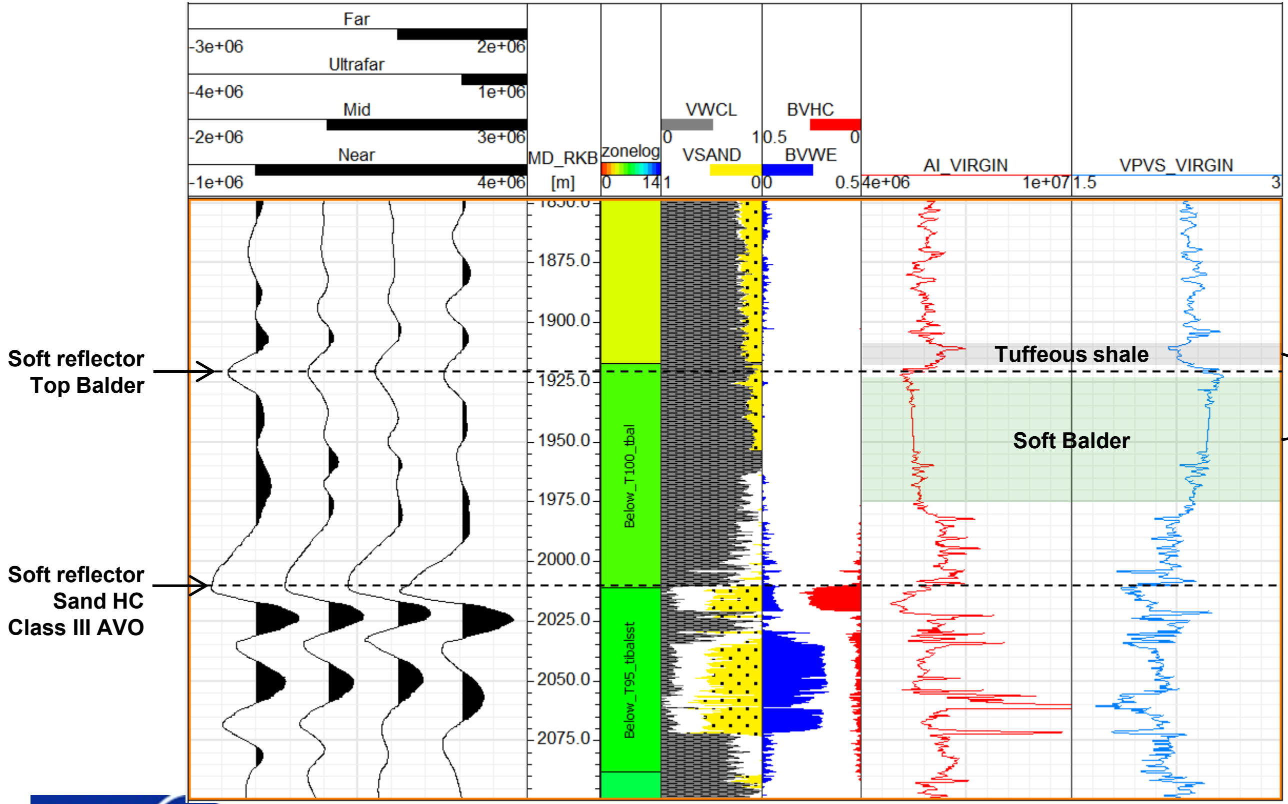
(from McKie et al. Geol. Soc. London Spec. Pub. 403, 2015)

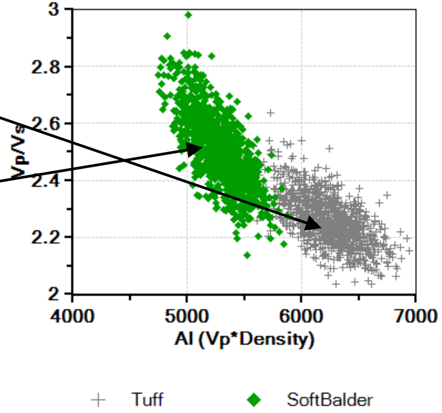
Identify LFCs from logs and well tops

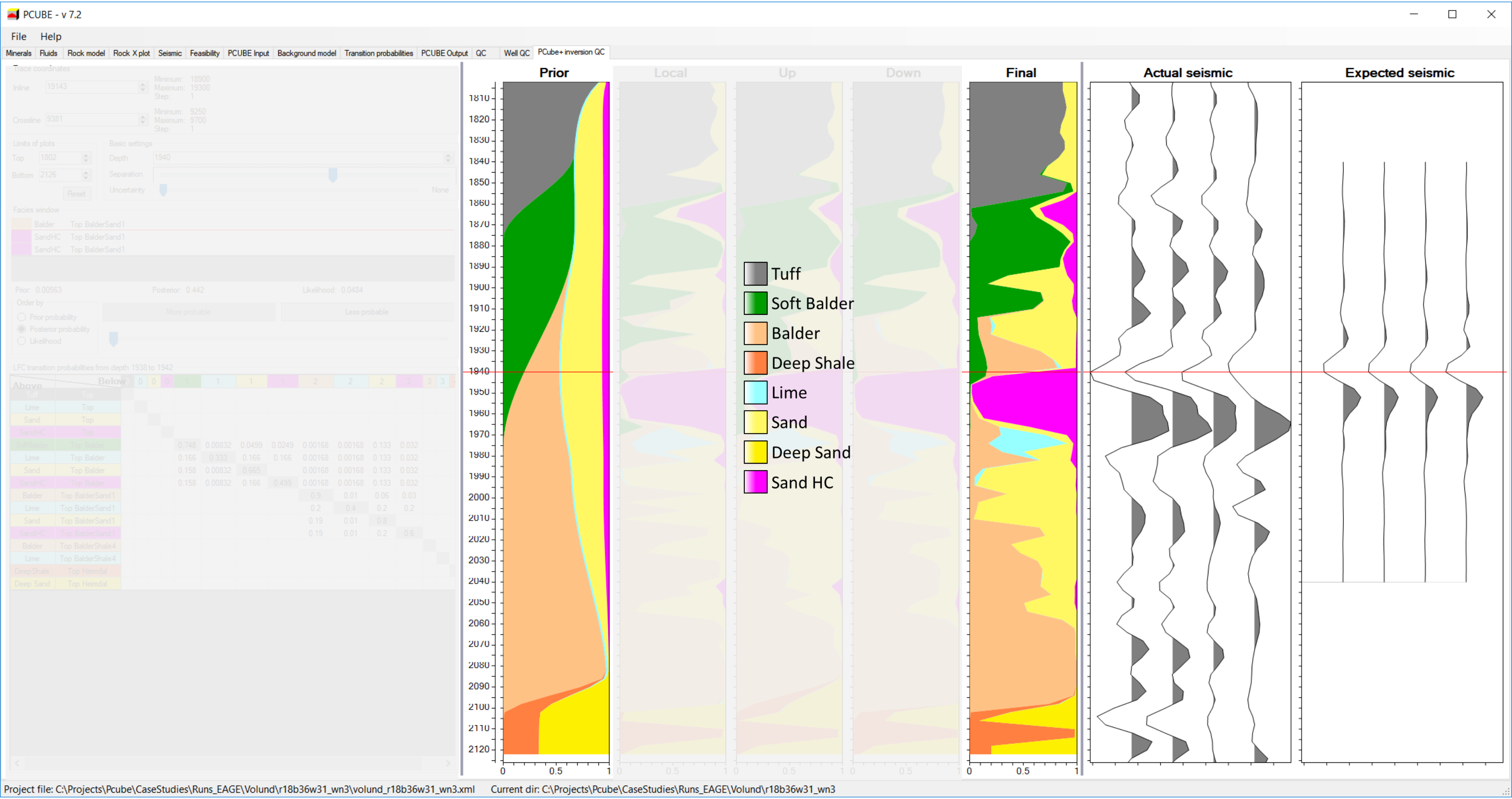




Define LFCs according to seismic respons







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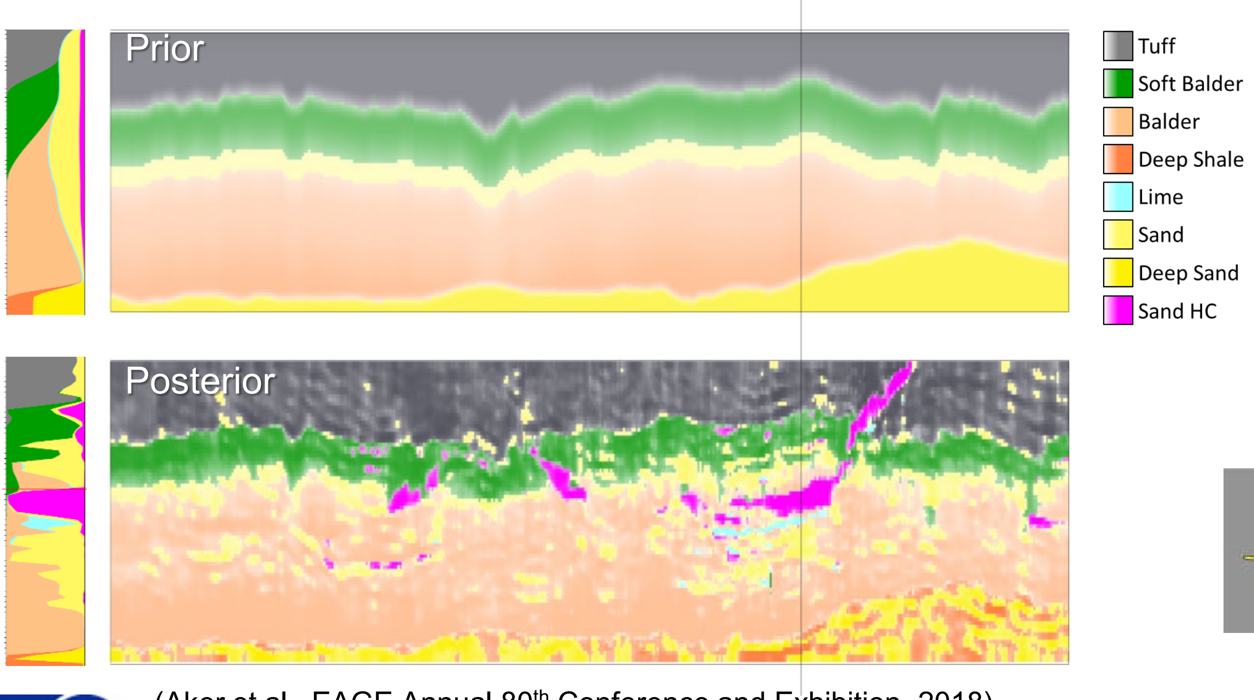
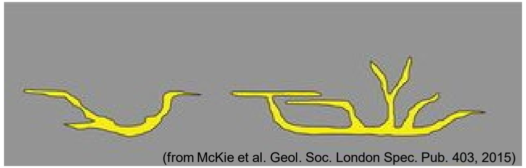
Cross section of most probable lithology fluid class

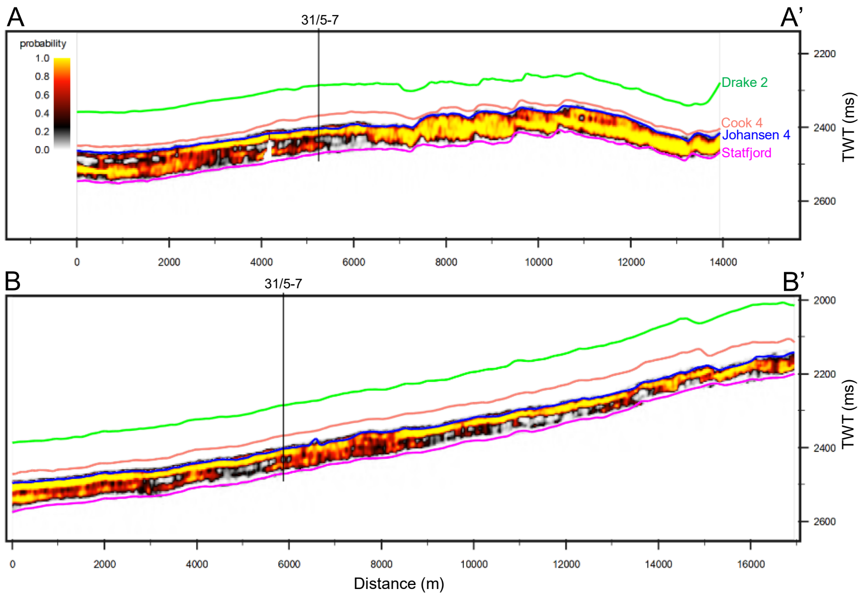




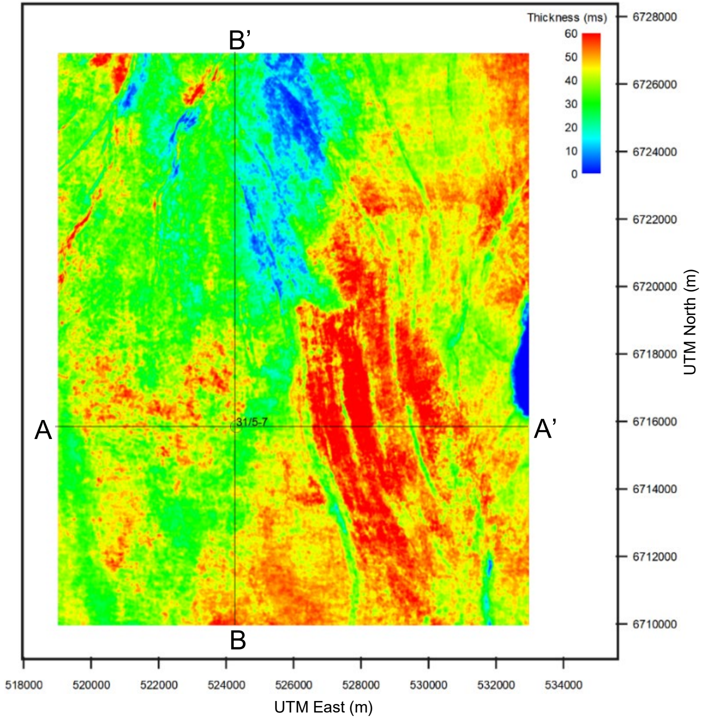
(Aker et al., EAGE Annual 80th Conference and Exhibition, 2018)

Probability and thickness estimates

Probability for Johansen sandstone



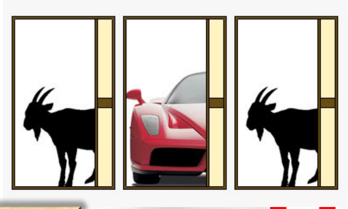
Expected thickness of Johansen sandstone



Probabilities quantify uncertainty

- Depend on assumptions
- Depend on approximations
- Uncertainty influences decisions *and* understanding
- Humans are familiar with uncertainty
- Humans easily jump to wrong conclusions

The Monty Hall problem



Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

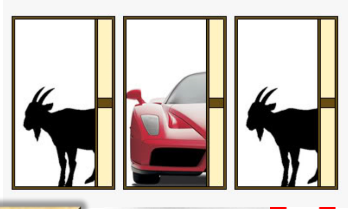
Monty Hall
(game show host)

Marilyn vos Savant
Marilyn in *Parade*
(IQ 228)

Ask Marilyn in *Parade*:
Contestants who switch have a 2/3 chance of winning the car,
while contestants who stick to their choice have only a 1/3 chance.

After the problem appeared in *Parade*, approximately 10 000 readers, including nearly 1 000 with PhDs, wrote to the magazine, most of them claiming Marilyn was wrong.

The Monty Hall problem



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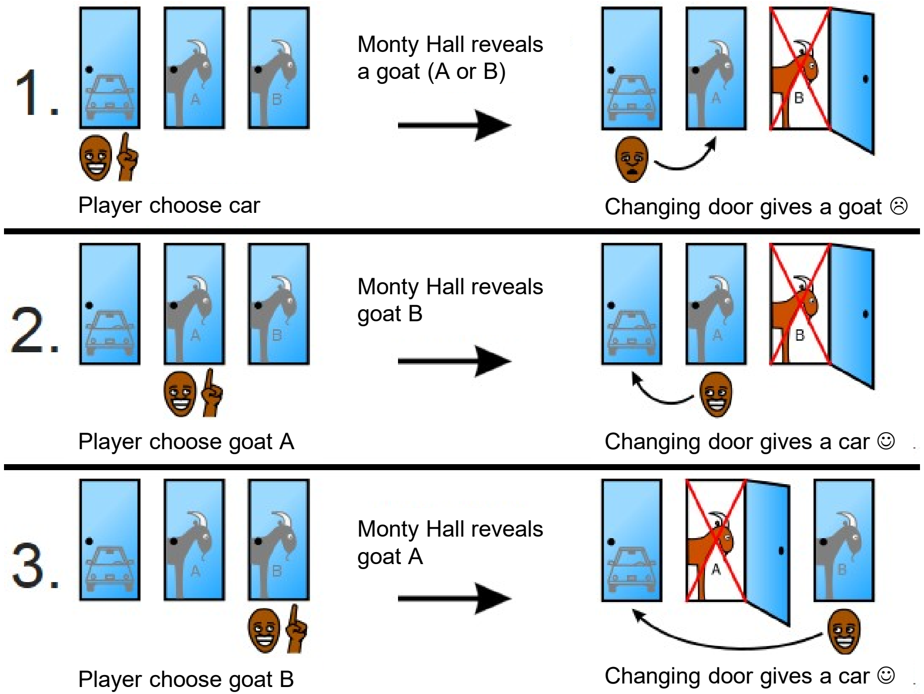
Monty Hall
(game show host)

Marilyn vos Savant
Marilyn in *Parade*
(IQ 228)

Can any of you make a good argument supporting Marilyn's claim?

Monty Hall


1.
Monty Hall reveals
a goat (A or B)
A
B
Player choose car
Changing door gives a goat ☹
2.
Monty Hall reveals
goat B
Player choose goat A
Changing door gives a car ☺
3.
Monty Hall reveals
goat A
Player choose goat B
Changing door gives a car ☺

I'll come straight to the point. In the following question and answer, you blew it!

"Suppose you're on a game show and given a choice of three doors. Behind one is a car; behind the others are goats. You pick Door No. 1, and the host, who knows what's behind them, opens No. 3, which has a goat. He then asks if you want to pick No. 2. Should you switch?"

You answered, "Yes. The first door has a 1/3 chance of winning, but the second has a 2/3 chance."

Let me explain: If one door is shown to be a loser, that information changes the probability to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and, in the future, being more careful.

—Robert Sachs, Ph.D., George Mason University, Fairfax, Va.

You blew it, and you blew it big! I'll explain: After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your answer or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!

—Scott Smith, Ph.D., University of Florida

Your answer to the question is in error. But if it is any consolation, many of my academic colleagues also have been stumped by this problem.

—Barry Pasternack, Ph.D., California Faculty Association

Good heavens! With so much learned opposition, I'll bet this one is going to keep math classes all over the country busy on Monday.

My original answer is correct. But first, let me explain why your answer is wrong. The winning odds of 1/3 on the first choice can't go up to 1/2 just because the host opens a losing door. To illustrate this, let's say we play a shell game. You look away, and I put a pea under one of three shells. Then I ask you to put your finger on a shell. The odds that your choice contains a pea are 1/3, agreed? Then I simply lift up an empty shell from the remaining two. As I can (and will) do this regardless of what you've chosen, we've learned nothing to allow us to revise the odds on the shell under your finger.

The benefits of switching are readily proved by playing through the six games that exhaust all the possibilities. For the first three games, you choose No. 1 and switch each time; for the second three games, you choose No. 1 and "stay" each time, and the host always opens a loser. Here are the results (each row is a game):

DOOR 1	DOOR 2	DOOR 3
AUTO	GOAT	GOAT
Switch and you lose.		
GOAT	**AUTO**	GOAT
Switch and you win.		
GOAT	GOAT	**AUTO**
Switch and you win.		
AUTO	GOAT	GOAT
Stay and you win.		
GOAT	**AUTO**	GOAT
Stay and you lose.		
GOAT	GOAT	**AUTO**
Stay and you lose.		

When you switch, you win two out of three times and lose one time in three; but when you don't switch, you only win one in three times.

You can play the game with another person acting as host with three playing cards—two jokers for the goats and an ace for the auto. Doing it a few hundred times to get valid statistics can get a little tedious, so perhaps you can assign it for extra credit—or for punishment. (*That'll* get their goats!)

If you have a question for Marilyn vos Savant, listed in the "Guinness Book of World Records Hall of Fame" for "Highest IQ," write: Ask Marilyn, Parade, 750 Third Ave., New York, N.Y. 10017. Because of volume of mail, personal replies are not possible.

PARADE MAGAZINE • DECEMBER 2, 1990 • PAGE 25

"In no other branch of mathematics is it so easy for experts to blunder as in probability theory."

Martin Gardner, writing about the related Three Prisoner Problem in *Scientific American* in 1959

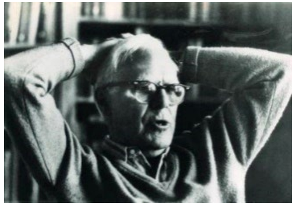


Image credit: Konrad Jacobs, Erlangen

You are in error—and you have ignored good counsel—but Albert Einstein earned a dearer place in the hearts of the people after he admitted his errors.
—Frank Rose, Ph.D., University of Michigan

I have been a faithful reader of your column and have not, until now, had any reason to doubt you. However, in this matter, in which I do have expertise, your answer is clearly at odds with the truth.
—James Rauff, Ph.D., Millikin University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?
—Charles Reid, Ph.D., University of Florida

Your logic is in error, and I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.
—W. Robert Smith, Ph.D. Georgia State University

You are utterly incorrect about the game-show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively toward the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?
—E. Ray Bobo, Ph.D., Georgetown University

I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.
—Kent Ford, Dickinson State University

Maybe women look at math problems differently than men.
—Don Edwards, Sunriver, Ore.

You are the goat!
—Glenn Calkins Western State College

You're wrong, but look at the positive side. If all those Ph.D.s were wrong, the country would be in very serious trouble.
—Everett Harman, Ph.D., U.S. Army Research Institute

Gasp! If this controversy continues, even the *postman* won't be able to fit into the mailroom. I'm receiving thousands of letters, nearly all insisting that I'm wrong, including one from the deputy director of the Center for Defense Information and another from a research mathematical statistician from the National Institutes of Health! Of the letters from the general public, 92% are against my answer; and of the letters from universities, 65% are against my answer. Overall, nine out of 10 readers completely disagree with my reply.

But math answers aren't determined by votes. For those readers new to all this, here's the original question and answer in full, to which the first readers responded:

"Suppose you're on a game show, and you're given a choice of three doors. Behind one door is a car; behind the others, goats. You pick a door—say, No.1—and the host, who knows what's behind the doors, opens another door—say, No. 3—which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to switch your choice?"

I answered, "Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened. Suppose there are a *million* doors, and you pick door No. 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door No. 777,777. You'd switch to that door pretty fast, wouldn't you?"

So many readers wrote to say they thought there was *no* advantage to switching (and that the chances became equal) that we published a second explanatory column, affirming the correctness of the original reply and using a shell game and a probability grid as illustrations.

Now we're receiving far *more* mail, and even newspaper columnists are joining in the fray. The day after the second column appeared, lights started flashing here at the magazine. Telephone calls poured into the switchboard, fax machines churned out copy, and the mailroom began to sink under its own weight. Incredulous at the response, we read wild accusations of intellectual irresponsibility and, as the days went by, we were even more incredulous to read embarrassed retractions from some of those same people!

The reaction is understandable. When reality clashes so violently with intuition, people are shaken.

But understanding is strength, so let's look at it again, remembering that the original answer defines certain conditions—the most significant of which is that *the host will always open a losing door on purpose*. (There's no way he can always open a losing door by chance!) Anything else is a different question.

The original answer is still correct, and the key to it lies in the question: *Should you switch?* Suppose we pause at that point, and a UFO settles down onto the stage. A little green woman emerges, and the host asks her to point to one of the two unopened doors. The chances that *she'll* randomly choose the one with the prize are 1/2. But that's because she lacks the advantage the *original* contestant had—the help of the host. (Try to forget any particular television show.)

When you first choose door No. 1 from among the three, there's a 1/3 chance that the prize is behind that one and a 2/3 chance that it's behind one of the others. *But then the host steps in and gives you a clue.* If the prize is behind No. 2, the host shows you No. 3; and if the prize is behind No. 3, the host shows you No. 2. So when you switch, you win if the prize is behind No. 2 *or* No. 3. *YOU WIN EITHER WAY!* But if you *don't* switch, you win only if the prize is behind door No. 1.

And as this problem is of such intense interest, I'm willing to put my thinking to the test with a nationwide experiment. This is a call to math classes all across the country. Set up a probability trial exactly as outlined below and send me a chart of all the games, along with a cover letter repeating just how you did it, so we can make sure the methods are consistent.

One student plays the contestant, another plays the host. Label three paper cups No. 1, No. 2 and No. 3. While the contestant looks away, the host randomly hides a penny under a cup by throwing a die until a 1, 2 or 3 comes up. Next, the contestant randomly points to a cup by throwing a die the same way. Then the host purposely lifts up a losing cup from the two unchosen. Last, the contestant "stays" and lifts up his original cup to see if it covers the penny. Play "not switching" 200 times and keep track of how often the contestant wins.

Then test the other strategy. Play the game the same way until the last instruction, at which point the contestant instead "switches" and lifts up the cup *not* chosen by anyone to see if it covers the penny. Play "switching" 200 times also.

And here's one last letter:

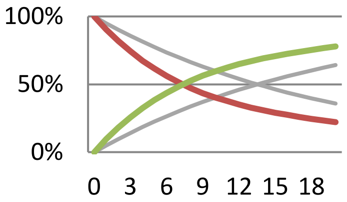
Dear Marilyn:
You are indeed correct. My colleagues at work had a ball with this problem, and I dare say that most of them—including me at first—thought you were wrong!
—Seth Kalson, Ph.D., Massachusetts Institute of Technology

Thanks, MIT. I needed that!

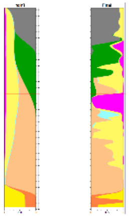
If you have a question for Marilyn vos Savant, who is listed in the "Guinness Book of World Records Hall of Fame" for "Highest IQ," send it to: Ask Marilyn, PARADE, 750 Third Ave., New York, N.Y. 10017. Because of volume of mail, personal replies are not possible.

NR

Decisions are based on (small) probabilities



Changes in probabilities has a huge impact

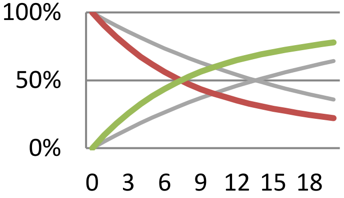


Uncertainties can be reduced and quantified

Doing the math correct is important

No amount of careful planning can beat pure luck

Decisions are based on (smal



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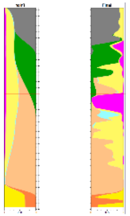

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